INSTITUT ZA RUDARSTVO I METALURGIJU BOR YU ISSN: 1451-0162
KOMITET ZA PODZEMNU EKSPLOATACIJU MINERALNIH SIROVINA UDK: 622

UDK: 622.03:681.3(045)=861

Gabrijela Popović, Dragiša Stanujkić*, Rodoljub Jovanović**

IZBOR RUDNOG LEŽIŠTA KOMBINOVANOM PRIMENOM TOPSIS I AHP METODE

Izvod

U radu je predložena metodologija izbora rudnog ležišta primenom TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) i AHP metode (Analytic Hierarchy Process). Predložena metodologija je zasnovana na primeni metoda višekriterijumskog odlučivanja, pri čemu je AHP metoda primenjena za određivanje težina kriterijuma. Efikasnost predložene metodologije prikazana je primenom realne studije slučaja.

Ključne reči: *Izbor rudnog ležišta; TOPSIS; AHP; MCDM: tehnički kriterijumi*

UVOD

Višekriterijumsko odlučivanje (multiple criteria decision making – MCDM) se generalno može opisati kao proces izbora jedne iz skupa raspoloživih alternativa, koja najefikasnije ispunjava postavljene ciljeve. Tokom poslednje dve decenije značajna pažnja je usmerena na razvoj i primenu MCDM metoda u rešavanju problema i dilema u oblasti upravljanja prirodnim resursima. Primeri se mogu naći u mnogim časopisima [1, 2, 3, 4, 5, 6, 7]. Značajan broj radova se bavi primenom ovih metoda u oblasti obnovljivih resursa i održivog razvoja [8, 9, 10]. Ali šta je sa primenom MCDM metoda u odlučivanju u mineralnoj industriji i metalurgiji? Neki autori su se bavili ovim pitanjima.

Izbor metode rudarenja je veoma česta tema u oblasti mineralnih resursa. Alpay i Yavuz su razmatrali sistem za podršku odlučivanju u izboru metode podzemnog rudarenja [11]. Naghadehi *et al.* su učinili nešto slično. Oni su predložili fazi AHP za određivanje optimalnog metoda rudarenja u iranskom rudniku [12]. Azimi *et al.* su rangirali strategije rudarskog sektora pomoću AHP i TOPSIS metode u okviru SWOT-a [13]. Samimi Namin *et al.* su predložili novi model, baziran na fazi odlučivanju, za izbor metoda rudarenja [14].

U ovom radu je predloženo korišćenje TOPSIS-a, između ostalih MCDM metoda, u preliminarnoj analizi za izbor odgovarajućeg rudnog ležišta za eksploataciju. AHP je upotrebljen za određivanje težina kriterijuma. Rad je organizovan na sledeći način: u drugom delu pažnja je usmerena na TOPSIS i AHP; u trećem delu je prikazan numerički primer koji je baziran na realnim podacima; a zatim sledi zaključak u četvrtom delu.

* *Fakultet za menadžment Zaječar, gabrijela.popovic@fmz.edu.rs,*

2. KOMBINACIJA TOPSIS I AHP METODE

TOPSIS su razvili Hwang i Yoon [15]. Navedena metoda je takođe prikazana u radu Chena i Hwanga [16], u kome je dat i osvrt na rad Hwanga i Yoona. TOPSIS nastoji da definiše idealno i anti-idealno rešenje [17]. Osnovni princip je da izabrana alternativa treba da ima najmanju distancu od idealnog rešenja.

AHP je predložio Saaty [18, 19] za oblikovanje subjektivnih procesa odlučivanja baziranih na više kriterijuma u hijerarhijskoj strukturi. Saaty je konstatovao da je najkreativniji zadatak u odlučivanju biranje faktora koji su važni za određenu odluku. U AHP-u donosilac odluke strukturiše izabrane faktore hijerarhijski, prema opadajućem redosledu od sveukupnog cilja do kriterijuma, sub-kriterijuma i alternativa na uzastopnim nivoima [20]. Ovaj model je takođe pogodan za determinisanje relativnih težina kriterijuma u odgovarajućoj hijerarhijskoj strukturi. Postoje brojne metode za determinisanje težina u AHP-u. Neke od njih su:

- metod aritmetičkih sredina normalizovanih kolona (average of normalized columns – ANC),
- metod normalizovanih suma redova (normalization of row average – NRA), i
- metod normalizovanih geometrijskih sredina redova (normalization of the geometric mean of the rows – NGM) [21].

TOPSIS i AHP su dokazano korisni za modeliranje i analizu različitih situacija u odlučivanju u raznim poljima nauke i tehnologije [15, 18]. Navedene metode su usmerene na rešavanje problema koji su vezani za izbor jedne alternative od više ponuđenih. AHP metod je pogodan za primenu i kod opipljivih i kod neopipljivih kriterijuma, naročito kada subjektive procene pojedinaca čine važan deo procesa odlučivanja. TOPSIS je prikladniji za opipljive kriterijume [22]. Sledeći koraci predstavljaju proceduru kombinovanog TOPSIS-a i AHP-a:

Korak 1. ***Determinisati cilj i evaluacione kriterijume.***

Korak 2. ***Prikazati u obliku matrice sve raspoložive informacije o kriterijumima.*** Tabela 1 predstavlja informacionu tabelu TOPSIS-a [23]:

Tabela 1. *Informaciona tabela TOPSIS-a*

Alterantive	***Kriterijumi***			
	C_1	C_2	...	C_j
A_1	x_{11}	x_{12}	...	x_{1j}
A_2	x_{21}	x_{22}	...	x_{2j}
. . .	. . .	. . .	. . .	. . .
A_i	x_{n1}	x_{n2}	...	x_{ij}
W	w_1	w_2	...	w_j

gde:

$A_1, A_2, ..., A_i$ predstavljaju moguće alternative između kojih donosilac odluke treba da bira,

$C_1, C_2, ..., C_j$ su kriterijumi kojima se vrednuju performanse alternativa,

x_{ij} je ocena alternative A_i u odnosu na kriterijum C_j,

w_j je težina kriterijuma C_j,

$i = 1,...,m,$ gde je m broj alternativa, a

$j = 1,...,n,$ gde je n broj kriterijuma [24].

Kriterijumi izraženi u kvalitatvnim pokazateljima treba da budu transformisani u kvantitativne pokazatelje. U tu svrhu se koristi numerička skala prikazana u Tabeli 2:

Tabela 2. *Transformisanje kvalitativnih u kvantitativne pokazatelje*

Kvalitativni pokazatelji	**Kvantitativni pokazatelji**	
	prihodni - max	**rashodni - min**
Veoma visok	9	1
Visok	7	3
Prosečan	5	5
Nizak	3	7
Veoma nizak	1	9

Korak 3. *Izračunati normalizovanu matricu odlučivanja.* Normalizovana vrednost r_{ij} se računa na sledeći način:

$$r_{ij} = x_{ij} / \sqrt{\sum_{i=1}^{m} x_{ij}^2} \qquad (1)$$

Korak 4. *Determinisati relativni značaj različitih kriterijuma u skladu sa postavljenim ciljem.* Neophodno je konstruisati matricu poređenja u parovima primenom fundamentalne skale AHP-a. Navedena skala je prikazana u Tabeli 3 [19]:

Tabela 3. *Fundamentalna skala AHP-a*

Evaluaciona skala	**Objašnjenje**
1	Jednaka važnost
3	Malo važnije
5	Mnogo važnije
7	Izuzetno važnije
9	Najvažnije
2, 4, 6, 8	Međuvrednosti predložene skale

Kriterijum koji se poredi sam sa sobom se uvek označava sa 1. Unosi u osnovnoj dijagonali matrice upoređivanja, takođe, uvek iznose 1.

Kada je izvršeno poređenje u parovima kriterijuma i sa kriterijumom j, potrebno je sačiniti matricu, u kojoj a_{ij} reprezentuje komparativnu prednost kriterijuma C_i u odnosu na kriterijum C_j. U matrici $a_{ij} = 1$ je kada je $i = j$ i $a_{ji} = a_{ij}$.

Tabela 4. *Matrica poređenja u parovima*

	C_1	C_2	C_3	$\dots$	C_j
C_1	a_{11}	a_{12}	a_{13}	$\dots$	a_{1j}
C_2	a_{21}	a_{22}	a_{23}	$\dots$	a_{2j}
. . .	. . .	. . .	. . .	. . .	. . .
C_j	a_{j1}	a_{j2}	a_{j3}	$\dots$	a_{jj}

Zatim treba izračunati normalizovanu težinu w_j svakog kriterijuma korišćenjem sledećih formula:

$$GM_i = \left(\prod_{i=1}^{n} a_{ij} \right)^{1/n}, \qquad (2)$$

$$w_j = GM_i \Big/ \sum_{i=1}^{n} GM_i \qquad (3)$$

Sledi determinisanje maksimalne sopstvene vrednosti matrice poređenja $\lambda_{\max}$ i izračunavanje indeksa konzistentnosti CI :

$$CI = (\lambda_{max} - n)/(n-1) \qquad (4)$$

Nakon toga, računa se slučajni indeks konzistenstnosti RI za kriterijume koji su korišćeni u procesu odlučivanja. U Tabeli 5 su prikazani navedeni indeksi:

Tabela 5. *Slučajni indeks konzistenstnosti*

Broj kriterijuma	1	2	3	4	5	6	7	8	9
Slučajni indeks konzistenstnosti	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45

Zatim se pristupa izračunavanju koeficijenta konzistentnosti CR pomoću sledeće formule:

$$CR = CR/RI \qquad (5)$$

Ako vrednost CR iznosi 0.1 ili je niža, smatra se prihvatljivom i odražava pravilno prosuđivanje.

Korak 5. ***Formirati težinski normalizovanu matricu odlučivanja.*** Težinski normalizovana vrednost v_{ij} se računa na sledeći način:

$$v_{ij} = w_j r_{ij} \qquad (6)$$

Korak 6. ***Determinisati idealno rešenje A^+ i anti-idealno rešenje A^-.*** Idealno A^+ i anti-idealno rešenje A^- se računa korišćenjem sledećih formula:

$$A^+ = \{v_1^+, \dots, v_n^+\} = \left\{ \left(\max_i v_{ij} \mid i \in I' \right), \left(\min_i v_{ij} \mid i \in I'' \right) \right\}, \qquad (7)$$

$$A^- = \{v_1^-, \dots, v_n^-\} = \left\{ \left(\min_i v_{ij} \mid i \in I' \right), \left(\max_i v_{ij} \mid i \in I'' \right) \right\}, \qquad (8)$$

gde se I' odnosi na skup prihodnih kriterijuma, a I'' na skup rashodnih kriterijuma.

Korak 7. ***Izračunati odstojanje svake alternative od idealnog rešenja D_i^+ i anti-idealnog rešenja D_i^- korišćenjem n-dimenzionalne Euklidove distance.*** Odstojanje D_i^+ i D_i^- se računa na sledeći način:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+ \right)}, \qquad (9)$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)}. \qquad (10)$$

Korak 8. ***Izračunati relativnu blizinu idealnom rešenju.*** Relativna blizina idealnom rešenju je definisana na sledeći način:

$$C_i^+ = D_i^- / \left(D_i^+ + D_i^-\right), \qquad (11)$$

gde je $0 \le C^+ \le 1$.

Ako je alternativa i bliža A^+, onda se C_i približava 1.

Korak 9. ***Rangirati alternative.*** Alternative se mogu rangirati po opadajućem redosledu C_i.

3. IZBOR RUDNOG LEŽIŠTA

Centralna Srbija raspolaže različitim prirodnim resursima čija eksploatacija može obezbediti prosperitet navedenom području. Među prirodnim resursima nalaze se i rudna ležišta koja nisu iskorišćena na pravi način. Osnovni metal koji se nalazi u ovim ležištima je bakar praćen određenim količinama srebra i zlata. Zaključeno je da će eksploatacija pravog rudnog ležišta, koje poseduje najbolje karakteristike, doprineti ekonomskom rastu u Centralnoj Srbiji. Upoređena su sledeća rudna ležišta:

- ***A_1*** **- Južni revir – opština Majdanpek**
- ***A_2*** **- Severni revir – opština Majdanpek**
- ***A_3*** **– Borska reka – opština Bor**
- ***A_4*** **- Cerovo – opština Bor**

Osnovno pitanje koje se nameće glasi: *Koje ležište treba da ima prvenstvo u eksploataciji?*

U razvoju sistema za podršku odlučivanju za evaluaciju i izbor rudnog ležišta korišćena je TOPSIS metoda, a za determinisanje težina kriterijuma AHP metoda. Kriterijumi na kojima se bazira višekriterijumska analiza su isključivo tehnički. Nije uzet u obzir uticaj koji eksploatacija ima na životnu sredinu, niti ekonomski aspekt eksploatacije. Kriterijumi su sledeći:

- ***C_1*****-Sadržaj bakra u rudi** (%). Prednost ima ležište čiji je sadržaj bakra u rudi veći.
- ***C_2*****-Sadržaj srebra u rudi** (g/t). Prednost ima ležište čiji je sadržaj srebra u rudi veći.
- ***C_3*****-Sadržaj zlata u rudi** (g/t). Prednost ima ležište čiji je sadržaj zlata u rudi veći.
- ***C_4*****-Ispitanost bilansnih rezervi.** Ispitane količine minerala u rudnom ležištu ili delu ležišta koje mogu biti racionalno i ekonomično iskorišćene pomoću postojeće tehnike i tehnologije za eksploataciju i preradu. Bolje ispitana ležišta imaju prednost.
- ***C_5*****-Lokacija.** Geoprostorni položaj rudnog ležišta i saobraćajna infrastruktura. Poželjno je ležište koje ima bolju poziciju i koje je bliže glavnim magistralama.
- ***C_6*****-Rudarsko-geološki uslovi eksploatacije.** Oni uključuju mnoge karakteristike rudnog ležišta, kao što su: oblik rudnog tela, dubina zaleganja, kontakt sa okolnim stenama, a koje su veoma značajne za izbor metoda eksploatacije i imaju uticaja na troškove eksploatacije.

Informacije o rudnim ležištima su preuzete iz studije koja predstavlja deo šire studije pod nazivom *Potencijali ekonomskog razvoja Timočke Krajine* i iz *Field guide-a*. [25, 26].

3.1. Primer

Primena kombinacije TOPSIS i AHP metode u rangiranju rudnih ležišta Centralne Srbije prikazana je na osnovu numeričkog primera. Tri eksperta su izvršila evaluaciju četiri rudna ležišta A_1, A_2, A_3 i A_4 na osnovu šest evaluacionih kriterijuma $C_1, C_2, C_3, ..., C_6$, koji su prethodno navedeni.

Tabela 6. *Sirovi podaci*

	Sadržaj bakra %	**Sadržaj srebra g/t**	**Sadržaj zlata g/t**	**Ispitanost bilansnih rezervi**	**Lokacija**	**Rudarsko-geološki uslovi eksploatacije**
	max	**max**	**max**	**max**	**min**	**max**
Južni revir	0.335	1.260	0.188	visok	srednji	visok
Severni revir	0.306	2.001	0.263	visok	srednji	visok
Borska reka	0.620	1.920	0.240	visok	vrlo nizak	vrlo nizak
Cerovo	0.340	1.800	0.110	visok	nizak	nizak

Kvalitativni podaci su prevedeni u kvantitativne primenom numeričke skale koja je prikazana u Tabeli 2.

Tabela 7. *Početna matrica odlučivanja*

	Kriterijumi					
	C_1	C_2	C_3	C_4	C_5	C_6
Alternative	Sadržaj bakra %	Sadržaj srebra g/t	Sadržaj zlata g/t	Ispitanost bilansnih rezervi	Lokacija	Rudarsko-geološki uslovi eksploatacije
	max	max	max	max	min	max
A_1	0.335	1.260	0.188	7	5	7
A_2	0.306	2.001	0.263	7	5	7
A_3	0.620	1.920	0.240	7	9	1
A_4	0.340	1.800	0.110	7	7	3

Tabela 8. *Normalizovana matrica odlučivanja*

	Kriterijumi					
Alternative	C_1	C_2	C_3	C_4	C_5	C_6
	max	**max**	**max**	**max**	**min**	**max**
A_1	0.3987	0.3561	0.4504	0.5000	0.3727	0.6736
A_2	0.3642	0.5656	0.6301	0.5000	0.3727	0.6736
A_3	0.7380	0.5427	0.5750	0.5000	0.6708	0.0962
A_4	0.4047	0.5088	0.2635	0.5000	0.5217	0.2887

Koraci 1, **2** i **3** su izvršeni.

Korak 4. U cilju determinisanja relativne važnosti svih mogućih parova kriterijuma uz poštovanje osnovnog cilja, konsultovana su tri eksperta iz oblasti rudarstva. Njihove ocene su date u matricama u vidu razlomaka (Tabela 9, Tabela 10 i Tabela 11). Za prikazivanje ocena je korišćena fundamentalna skala AHP-a, prikazana u Tabeli 3. Relativne normalizovane težine w_j svakog kriterijuma j su izračunate primenom formula (2) i (3). Koeficijent konzistentnosti CR je proveren pomoću formula (4) i (5).

Tabela 9. *Matrica poređenja u parovima - Ekspert 1*

	C_1	C_2	C_3	C_4	C_5	C_6	w_j
C_1	1	9	1	1	7	1	0.233
C_2	1/9	1	1/9	1/9	1/7	1/7	0.020
C_3	1	9	1	1	7	1/3	0.194
C_4	1	9	1	1	5	1/3	0.183
C_5	1/7	7	1/7	1/5	1	1/7	0.047
C_6	1	7	3	3	7	1	0.322
				$CR = 9.96\%$			

Tabela 10. *Matrica poređenja u parovima - Ekspert 2*

	C_1	C_2	C_3	C_4	C_5	C_6	w_j
C_1	1	5	3	1/7	1/5	1/3	0.079
C_2	1/5	1	1/3	1/9	1/9	1/7	0.024
C_3	1/3	3	1	1/9	1/7	1/5	0.042
C_4	7	9	9	1	3	1	0.376
C_5	5	9	7	1/3	1	1/3	0.197
C_6	3	7	5	1	3	1	0.284
				$CR = 8.64\%$			

Tabela 11. *Matrica poređenja u parovima - Expert 3*

	C_1	C_2	C_3	C_4	C_5	C_6	w_j
C_1	1	9	3	5	7	1/3	0.271
C_2	1/9	1	1/7	1/5	1/3	1/9	0.023
C_3	1/3	7	1	3	5	1/5	0.144
C_4	1/5	5	1/3	1	3	1/5	0.079
C_5	1/7	3	1/5	1/3	1	1/9	0.040
C_6	3	9	5	5	9	1	0.443
				$CR = 7.53\%$			

Tri različita suda i zbog toga tri različite težine su svedene na zajedničku primenom formule (2). Dobijene težine, prikazane u Tabeli 12, su normalizovane primenom formule (3) zbog toga što ukupan rezultat nije iznosio 1.

Korak 5. Težinski normalizovana matrica odlučivanja je dobijena primenom formule (6) i prikazana je u Tabeli 12.

Tabela 12. *Težinski normalizovana matrica odlučivanja*

	Kriterijumi					
	C_1	C_2	C_3	C_4	C_5	C_6
Težine	0.192	0.025	0.118	0.198	0.080	0.386
Alterantive	max	max	max	max	min	max
A_1	0.0766	0.0089	0.0531	0.0990	0.0298	0.2600
A_2	0.0699	0.0141	0.744	0.0990	0.0298	0.2600
A_3	0.1417	0.0136	0.0679	0.0990	0.0537	0.0371
A_4	0.0777	0.0127	0.0311	0.0990	0.0417	0.1114

Korak 6. Idealno A^+ i anti-idealno rešenje A^-, prikazano u Tabeli 13, je izračunato putem formula (7) i (8).

Tabela 13. *Idealno A^+ i anti-idealno rešenje A^-*

A^+	0.1417	0.0141	0.0744	0.0990	0.0298	0.2600
A^-	0.0699	0.0089	0.0311	0.0990	0.0537	0.0371

Korak 7. Odstojanja D_i^+ i D_i^- su određena primenom formula (9) i (10). Rezultati su prikazani u Tabeli 14, I i II kolona.

Korak 8. Relativna blizina određenog rešenja idealnom C_i je izračunata pomoću formule (11) i prikazana je u Tabeli 14, III kolona.

Tabela 14. *Odstojanje i relativna blizina idealnom rešenju*

Alternative	D_i^+	D_i^-	C_i
	I	**II**	**III**
A_1	0.0687	0.2253	0.7663
A_2	0.0718	0.2283	0.7609
A_3	0.2242	0.0808	0.2648
A_4	0.1679	0.0757	0.3109

Korak 9. U skladu sa dobijenim rezultatima, rangiranje je sledeće (Tabela 15):

Tabela 15. *Rang*

Alternative	C_i	**Rang**
A_1	0.7663	1
A_2	0.7602	2
A_3	0.2648	4
A_4	0.3109	3

4. ZAKLJUČAK

Konačni rezultati pokazuju da je rudno ležište Južni revir ispravan izbor u postojećim uslovima. Za njim sledi Severni revir, treći je Cerovo, a na poslednjem mestu se nalazi Borska reka. Kao što je ranije rečeno, u obzir su uzeti samo tehnički aspekti i to: sadržaj određenog metala, ispitanost bilansnih rezervi, lokacija i rudarsko-geološki uslovi eksploatacije, te su dobijeni rezultati vezani isključivo za te kriterijume.

Predložena metodologija, zasnovana na TOPSIS i AHP metodi, pomaže u izboru rudnog ležišta i može biti korisna u preliminarnoj analizi. Metodologija može obuhvatiti bilo koji broj kriterijuma i nudi objektivniji, jednostavniji i konzistentniji pristup izboru rudnog ležišta. Ova metodologija se može primeniti u evaluaciji i rangiranju različitih skupova alternativnih rudnih ležišta. Takođe, izbor rudnih ležišta se može zasnivati na različitim kriterijumima, ne samo na tehničkim aspektima eksploatacije.

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MINING AND METALLURGY INSTITUTE BOR YU ISSN: 1451-0162
COMMITTEE OF UNDERGROUND EXPLOITATION OF THE MINERAL DEPOSITS UDK: 622

UDK: 622.03:681.3(045)=20

Gabrijela Popovic, Dragisa Stanujkic*, Rodoljub Jovanovic**

ORE DEPOSIT SELECTION USING THE COMBINED TOPSIS AND AHP METHOD

Abstract

This paper proposes a methodology for ore deposit selection using TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) and AHP method (Analytic Hierarchy Process). The proposed methodology is based on the use of multi-criteria decision making methods, where AHP method is used to determine the criteria weights. The efficiency of proposed methodology is shown using the real case study.

Keywords: *ore deposit selection, TOPSIS, AH, MCDM, technical criteria*

1. INTRODUCTION

The multiple criteria decision making (MCDM) can be generally described as the process of selecting one from a set of available alternatives, which meets the most efficiently the set objectives of choice. Over the past two decades, a significant attention has been focused on development and using the MCDM methods in resolving problems and dilemmas in the field of natural resource management. Examples can be found in many journals [1, 2, 3, 4, 5, 6, 7]. There is a significant number of papers about using these methods in the field of renewable resources and sustainable development [8, 9, 10]. But, what about use the MCDM methods in making decisions in mineral industry and metallurgy? Some authors have studied these issues.

In the field of mineral resources, a selection of mining method by MCDM is a very common theme. Alpay and Yavuz considered a decision support system for underground mining method selection [11]. Naghadehi *et al.* did something similar. They proposed fuzzy analytic hierarchy process in determining an optimum mining method for mine in Iran [12]. Azimi *et al*. ranked strategies of mining sector through AHP and TOPSIS in a SWOT framework [13]. Samimi Namin *et al*. proposed a new model for mining method selection of mineral deposit, based on fuzzy decision making [14].

In this paper, TOPSIS is proposed, among other MCDM methods, for the use in preliminary analysis in selection the appropriate ore deposit for exploitation. AHP is used in determining the weights of criteria. The paper is organized as follows: in section 2, the attention is paid to TOPSIS and AHP; in section 3, a numerical example is given, based on real data; and section 4 contains conclusions.

* *Faculty of Management Zajecar, gabrijela.popovic@fmz.edu.rs*

2. COMBINATION OF TOPSIS AND AHP METHOD

TOPSIS is developed by Hwang and Yoon [15]. This method is also present in a work by Chen and Hwang [16], with reference to Hwang and Yoon. TOPSIS attempts to define the ideal and anti-ideal solution [17]. The main principle is that the chosen alternative should have the shortest distance from the ideal solution.

AHP was proposed by Saaty [18, 19] to model subjective decision-making processes, based on multiple criteria in a hierarchical system. Saaty said that the most creative task in a decision making is to choose the important factors for that decision. In AHP, a decision maker arranges these selected factors in a hierarchic structure descending from an overall goal to the criteria, sub-criteria and alternatives in successive levels [20]. This model is also convenient to determine the relative weights according to the appropriate hierarchical system. There are numerous methods for determining the weights in AHP. Three of them are:

- average of normalized columns (ANC),
- normalization of row average (NRA), and
- normalization the geometric mean of t rows (NGM) [21].

TOPSIS and AHP are logically proved to be useful for modeling and analyzing various types of decision making situations in numerous fields of science and technology [15, 18]. These methods are attached to solving a problem which is connected to selection an alternative from a number of alternatives. The AHP can deal with tangible as well as non-tangible criteria, especially where subjective judgments of different individuals constitute an important part of a decision process. The TOPSIS is more appropriate for tangible criteria [22]. Following steps represent combined TOPSIS and AHP procedure:

Step 1. ***Determine a goal and evaluation criteria***

Step 2. ***Represent in a matrix form all available information about the criteria.*** Table 1 represents TOPSIS information table [23]:

Table 1 *Information table of TOPSIS*

Alternatives	***Criteria***			
	C_1	C_2	…	C_j
A_1	x_{11}	x_{12}	…	x_{1j}
A_2	x_{21}	x_{22}	…	x_{2j}
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
A_i	x_{n1}	x_{n2}	…	x_{ij}
W	w_1	w_2	…	w_j

where:

$A_1, A_2, ..., A_i$ represent possible alternatives among which a decision maker has to choose,

$C_1, C_2, ..., C_j$ are criteria with which alternative performance are measured,

x_{ij} is rating of alternative A_i with respect to the criteria C_j,

w_j is the weight of the criteria C_j,

$i = 1,...,m,$ m is number of alternatives, and

$j = 1,...,n,$ n is number of criteria [24].

When some of criteria are shown as the qualitative values, they need to be changed into quantitative values. For that purpose. a numerical scale, shown in Table 2, is used:

Table 2. *Transformation of linguistic scales into quantitative values*

Linguistic scale	Quantitative value	
	benefit - max	cost - min
Very high	9	1
High	7	3
Average	5	5
Low	3	7
Very low	1	9

Step 3. ***Calculate a normalized decision matrix***. The normalized value r_{ij} is calculated as:

$$r_{ij} = x_{ij} / \sqrt{\sum_{i=1}^{m} x_{ij}^2} \; . \qquad (1)$$

Step 4. ***Determine the relative importance of different criteria with respect to the following goal.*** Construct a pairwise comparison matrix using the fundamental scale of AHP. This scale is present in Table 3 [19]:

Table 3. *Fundamental scale of AHP*

Evaluation scale	Definition
1	Equally important
3	Slightly more importance
5	Strongly more importance
7	Demonstrably more importance
9	Absolutely more importance
2, 4, 6, 8	Medium value of adjacent scale

A criterion, compared by itself, is always assigned with 1. The main diagonal entries of comparison matrix also are always 1.

When the pairwise comparison of criterion i with criterion j yields is performed, then there is a need for a square matrix, where a_{ij} denotes the comparative importance of criterion C_i with respect to criterion C_j. In the matrix $a_{ij} = 1$, when $i = j$ and $a_{ji} = a_{ij}$.

Table 4. Pairwase comparison matrix

	C_1	C_2	C_3	...	C_j
C_1	a_{11}	a_{12}	a_{13}	...	a_{1j}
C_2	a_{21}	a_{22}	a_{23}	...	a_{2j}
.	.	.	.	.	.
C_j	a_{j1}	a_{j2}	a_{j3}	...	a_{jj}

Calculate relative normalized weight w_j of each criterion using the following formulae:

$$GM_i = \left(\prod_{i=1}^{n} a_{ij} \right)^{1/n}, \quad (2)$$

$$w_j = GM_i / \sum_{i=1}^{n} GM_i . \quad (3)$$

Then, determine maximum eigenvalue λ_{max} and calculate the consistency index CI :

$$CI = (\lambda_{max} - n)/(n-1). \quad (4)$$

After that, the random index RI , for the number of criteria used in the decision making, has to be obtained. Table 5 shows this index:

Table 5. Random index details

Number of Criteria	1	2	3	4	5	6	7	8	9
Random Index	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45

Then, the consistency ratio CR is calculated using the following formula:

$$CR = CR/RI . \quad (5)$$

If value of CR is 0.1 or less than it is considered as acceptable, and it reflects an appropriate judgment.

Step 5. ***Create the weighted normalized decision matrix.*** The weighted normalized value v_{ij} is calculated as:

$$v_{ij} = w_j r_{ij} \quad (6)$$

Step 6. ***Determine ideal solution A^+ and anti-ideal solution A^-.*** The ideal solution A^+ and anti-ideal solution A^- are calculated using the following formulae:

$$A^+ = \{v_1^+, ..., v_n^+\} = \left\{ \left(\max_i v_{ij} \mid i \in I' \right), \left(\min_i v_{ij} \mid i \in I'' \right) \right\}, \quad (7)$$

$$A^- = \{v_1^-, ..., v_n^-\} = \left\{ \left(\min_i v_{ij} \mid i \in I' \right), \left(\max_i v_{ij} \mid i \in I'' \right) \right\}, \quad (8)$$

where I' is associated to a set of benefit criteria, and I'' is associated to a set of cost criteria.

Step 7. ***Calculate the separation of each alternative from ideal solution D_i^+, and anti-ideal solution D_i^- using the n-dimensional Euclidean distance.*** The ideal solution D_i^+ and anti-ideal solution D_i^- are calculated as:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+ \right)}, \quad (9)$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^- \right)}. \quad (10)$$

Step 8. ***Calculate the relative closeness to the ideal solution.*** The relative closeness to the ideal solution is defined as:

$$C_i^+ = D_i^- \Big/ \left(D_i^+ + D_i^-\right), \qquad (11)$$

where $0 \le C^+ \le 1$.

If an alternative i is closer to A^+, than C_i approaches to 1.

Step 9. ***Rank the preference order.*** A set of alternatives can be preferentially ranked according to the descending order of C_i.

3. ORE DEPOSIT SELECTION

Central Serbia has various natural resources whose exploitation could bring prosperity to this area. Among these natural resources, there are ore deposits, which are not properly used. The main metal in these deposits is copper including some amount of silver and gold. It is concluded that exploitation of the right ore deposit with the best characteristics, would lead to the economic growth of Central Serbia. The compared ore deposits are:

- **A_1 – South Mining District – Majdanpek municipality**
- **A_2 – North Mining District - Majdanpek municipality**
- **A_3 - Borska reka - Bor municipality**
- **A_4 - Cerovo - Bor municipality**

So, the main question is: *Which of these deposits should have priority in exploitation?*

TOPSIS method is used for development a decision support system for evaluation and selection the ore deposit, and AHP method for determining the weights of criteria. The criteria, under which the multi-criteria analysis is performed, are only technical. The effect of exploitation on the environment, as well as the economic aspect of exploitation, is not taken into account. The criteria are following:

- **C_1-Copper content in the ore** (%). The ore deposit with higher copper content has the advantage.
- **C_2-Silver content in the ore** (g/t). The ore deposit with higher silver content has the advantage.
- **C_3-Gold content in the ore** (g/t). The ore deposit with higher gold content has the advantage.
- **C_4-Testing the balance reserves.** Tested amounts of minerals in the ore deposit or a part of deposit, which can be rationally and economically recovered using the present techniques and technologies of exploitation and processing. Better tested deposits have the advantage.
- **C_5-Location.** Geospatial position of the ore deposit and transport infrastructure. The ore deposit which has better position and which is closer to the major highways is desirable.
- **C_6-Mining-geological conditions of exploitation.** Mining-geological conditions include many characteristics of ore deposit, such as: shape of ore body, depth, contact with the surrounding rocks and others that are very significant for selection the exploitation method and have influence on the exploitation costs.

Information about compared ore deposits are taken from the study that is a part of the extensive study *Potentials of Economic Development of the Timok Region* and from *Field Guide* [25,26].

3.1. Example

A numerical example is illustrated for presentation the combined TOPSIS and AHP for ranking the ore deposits in Central Serbia. Four ore deposits A_1, A_2, A_3 and A_4 are evaluated by three experts against six evaluating criteria $C_1, C_2, C_3, ..., C_6$, which are above mentioned.

Table 6. Raw data

	Copper content %	Silver content g/t	Gold content g/t	Testingof balance reserves	Location	Mining-geological conditions of exploitation
	max	max	max	max	min	max
S.M.District	0.335	1.260	0.188	high	average	high
N.M.District	0.306	2.001	0.263	high	average	high
Borska reka	0.620	1.920	0.240	high	very low	very low
Cerovo	0.340	1.800	0.110	high	low	low

Qualitative data are changed into quantitative using the numerical scale, shown in the Table 2.

Table 7. Initial decision matrix

	Criteria					
	C_1	C_2	C_3	C_4	C_5	C_6
Alternatives	Copper content %	Silver content g/t	Gold content g/t	Testing of balance reserves	Location	Mining-geological conditions of exploitation
	max	max	max	max	min	max
A_1	0.335	1.260	0.188	7	5	7
A_2	0.306	2.001	0.263	7	5	7
A_3	0.620	1.920	0.240	7	9	1
A_4	0.340	1.800	0.110	7	7	3

Normalized decision matrix (Table 8) is calculated by the formula (1).

Table 8. Normalized decision matrix

	Criteria					
Alternatives	C_1	C_2	C_3	C_4	C_5	C_6
	max	max	max	max	min	max
A_1	0.3987	0.3561	0.4504	0.5000	0.3727	0.6736
A_2	0.3642	0.5656	0.6301	0.5000	0.3727	0.6736
A_3	0.7380	0.5427	0.5750	0.5000	0.6708	0.0962
A_4	0.4047	0.5088	0.2635	0.5000	0.5217	0.2887

Note that **Steps 1**, **2** and **3** are done.

Step 4. In order to determine the relative importance of all possible pairs of criteria with respect to the overall goal, three experts in the field of mining are consulted. Their judgments are arranged into matrixes. The pairwise comparison judgment on the criteria is shown as the equation in all three matrixes (Table 9, Table 10, Table 11). The fundamental scale of AHP, shown in Table 3, is used in entering the judgments. The relative normalized weights w_j of each criterion j is calculated by the formulae (2) and (3). The consistency ratio CR is checked by the formulae (4) and (5).

Table 9. *Pairwise matrix - Expert 1*

	C_1	C_2	C_3	C_4	C_5	C_6	w_j
C_1	1	9	1	1	7	1	0.233
C_2	1/9	1	1/9	1/9	1/7	1/7	0.020
C_3	1	9	1	1	7	1/3	0.194
C_4	1	9	1	1	5	1/3	0.183
C_5	1/7	7	1/7	1/5	1	1/7	0.047
C_6	1	7	3	3	7	1	0.322

$CR = 9.96\%$

Table 10. *Pairwise matrix - Expert 2*

	C_1	C_2	C_3	C_4	C_5	C_6	w_j
C_1	1	5	3	1/7	1/5	1/3	0.079
C_2	1/5	1	1/3	1/9	1/9	1/7	0.024
C_3	1/3	3	1	1/9	1/7	1/5	0.042
C_4	7	9	9	1	3	1	0.376
C_5	5	9	7	1/3	1	1/3	0.197
C_6	3	7	5	1	3	1	0.284

$CR = 8.64\%$

Table 11. *Pairwise matrix - Expert 3*

	C_1	C_2	C_3	C_4	C_5	C_6	w_j
C_1	1	9	3	5	7	1/3	0.271
C_2	1/9	1	1/7	1/5	1/3	1/9	0.023
C_3	1/3	7	1	3	5	1/5	0.144
C_4	1/5	5	1/3	1	3	1/5	0.079
C_5	1/7	3	1/5	1/3	1	1/9	0.040
C_6	3	9	5	5	9	1	0.443

$CR = 7.53\%$

Three different judgments and therefore, different weights, are reduced to a common weight by the formula (2). Calculated weights are normalized by the formula (3) because the total result was not 1, and they are as in Table 12.

Step 5. The weighted normalized decision matrix is calculated by the formula (6) and shown in Table 12.

Table 12. *Weighted normalized decision matrix*

	Criteria					
	C_1	C_2	C_3	C_4	C_5	C_6
Weights	0.192	0.025	0.118	0.198	0.080	0.386
Alterantives	max	max	max	max	min	max
A_1	0.0766	0.0089	0.0531	0.0990	0.0298	0.2600
A_2	0.0699	0.0141	0.744	0.0990	0.0298	0.2600
A_3	0.1417	0.0136	0.0679	0.0990	0.0537	0.0371
A_4	0.0777	0.0127	0.0311	0.0990	0.0417	0.1114

Step 6. The ideal A^+ and anti-ideal solutions A^- are determined by the for mulae (7) and (8), and they are as in Table 13.

Table 13. *The ideal A^+ and anti-ideal solutions A^-*

A^+	0.1417	0.0141	0.0744	0.0990	0.0298	0.2600
A^-	0.0699	0.0089	0.0311	0.0990	0.0537	0.0371

Step 7. The separation measures D_i^+ and D_i^- are determined by using the formulae (9) and (10). The results are shown in Table 14, columns I and II.

Step 8. Relative closeness of particular solution to the ideal solution C_i is calculated by the formula (11), and it is given in Table 14, column III.

Table 14. *The separation measures and relative closeness to the ideal solution*

Alternative	D_i^+	D_i^-	C_i
	I	**II**	**III**
A_1	0.0687	0.2253	0.7663
A_2	0.0718	0.2283	0.7609
A_3	0.2242	0.0808	0.2648
A_4	0.1679	0.0757	0.3109

Step 9. According to the results, the rank is the following (Table 15):

Table 15. Ranking results

Alternative	C_i	Rank
A_1	0.7663	1
A_2	0.7602	2
A_3	0.2648	4
A_4	0.3109	3

4. CONCLUSION

The final results show that the ore deposit South Mining District is the appropriate choice for the given conditions. The second choice is the North Mining District, the third is Cerovo, and the last one is Borska reka. As it was discussed previously, only technical aspects were taken into consideration, such as content of certain metals, testing the balance reserves, location, and mining-geological conditions of exploitation and the results were obtained according to that criteria.

The proposed methodology based on TOPSIS and AHP helps in the ore deposit selection and it is useful in the preliminary analysis. The methodology can consider any number of criteria and offers more objective, simple and consistent approach to the ore deposit selection. This methodology could be used in evaluating and ranking any given set of alternative ore deposits. It can be used for the ore deposit selection according to different criteria, not only to technical aspects of exploitation.

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